REVIEWS

Vortex Dynamics. By P. G. SAFFMAN. Cambridge University Press, 1992. 311 pp. £35 or $69.95.

Any specialist in fluid dynamics today could describe the motion of a homogeneous fluid in terms of advection and diffusion of vorticity. Many papers have 'vorticity' and/or 'vortex' as key words in their titles. However, the number of monographs on the subject is surprisingly low. In the whole history of fluid mechanics there are only a few entirely devoted to vortex dynamics. Four of them are the books by H. Poincaré (1893), H. Villat (1930), C. Truesdell (1954) and M. A. Goldshtik (1981). Probably the well-known monograph by H. P. Greenspan (1968) could be included in this list, although he did not present his book from this point of view. Professor Saffman has cited all these books (except that by M. A. Goldshtik, which is published only in Russian), but a brief consideration shows that the character of the material included in these various monographs appears to be essentially different in each case. What is the reason why there are so few books on vortex dynamics and such different interpretations of the subject? To answer this question it is important to define the meaning of vortex dynamics. Professor Saffman does this as follows: 'The emphasis in this monograph is on the classical theory of inviscid incompressible fluids containing finite regions of vorticity. ... However, this volume focuses on those aspects of fluid motion which are primarily controlled by the vorticity and are such that the effects of the other fluid properties are secondary'. After a careful analysis of this definition it is clear that an enormous amount of pertinent material is not included in the book. The author himself points out that he has not included some material on boundary layers. To this may be added almost all plane fluid mechanics of ideal fluids, the theory of rotating fluids, the theory of Beltrami flows, a significant part of stability theory, and so on. The requirement 'finite regions of vorticity' is particularly restrictive.

Perhaps the reason for this situation and the answer to the questions we have asked are that from one point of view all hydrodynamics is included in vortex dynamics! However, composing a definition for different branches of science is a thankless task and Professor Saffman has not tried to do so. He has used his wide experience in research and in the teaching of vortex dynamics to select the material which in his opinion forms the core of the subject, and so he is able to present it briefly, simply and clearly.

The book can be divided into three parts according to subject matter: (i) a general, elementary introduction to the theory of fluid motion with vorticity (chapters 1–3); (ii) the theory of flows with concentrated vortices (chapters 6–12); (iii) a short introduction to a number of interesting and active fields in vortex dynamics (chapters 4–5, 13–14).

The first part is written according to the principle 'only the basic facts and nothing extra' and may certainly be recommended to all beginners in the subject. Chapter 3 on vortex momentum gives for the first time a detailed introduction to the subject, which often causes confusion.

The heart of the monograph is the second part, the theory of flows with concentrated vortices. There is no doubt that research in this class of flows forms the basis of vortex dynamics and is the reason for its existence as a separate branch of fluid dynamics. It is useful to formulate a definition of such flows as follows: a flow, in some parts of which (the vortex cores) the magnitude of the vorticity is much greater than any component of the rate-of-strain tensor, is a flow with concentrated vortices. This definition automatically excludes many other flows with large vorticity such as boundary layers. Alternatively, instead of a definition, it is possible to list the types of flows with concentrated vortices: line vortices, vortex patches, vortex rings, vortex filaments, etc., as does Professor Saffman. His book occupies a special place among the books mentioned above, because for the first time in the history of fluid dynamics the theory of such flows is systematically considered.

The second part of the book begins with the consideration of dynamics of singular distributions of vorticity (point vortices, vortex sheets). A large amount of literature has been published on the subject, to which Professor Saffman gives a very good guide; he includes such topics as the stability of the Kármán vortex street and the statistical mechanics of assemblies of line vortices. In chapter 6 the theory of vortex sheets is considered, and this is applied in chapter 8 to models of roll-up of a semi-infinitive vortex sheet, which is the basic mechanism of the creation of concentrated vortices in a homogeneous fluid. This is a masterly presentation of classical results as well as recent developments due to D. W. Moore, D. I. Pullin and others. The theory of straight-line vortices is presented in chapter 7. Professor Saffman gives a good guide to the associated large literature of this subject and includes discussions of such topics as the stability of a Kármán vortex street. The classic theory of the motion of axisymmetric vortex rings is given in chapter 10. Here, in particular, we have the first systematic presentation of a method of obtaining the analytic form for the velocity of propagation of vortex rings when the distribution of vorticity in the core is arbitrary. Chapters 9, 11 and 12 are devoted to modern developments. Here we have an unusually short and clear presentation of such questions as contour dynamics for two-dimensional patches of vorticity, the intriguing phenomenon of filamentation, the review of plane exact solutions, the cut-off method for the dynamics of a vortex filament, and the parametric (resonant) instability for vortices with elliptical cross-section of the core. The presentation of these chapters is unique, and is the only systematic source of information for anyone interested in the subject.

The third part of the book (chapters 4, 5, 13 and 14) is a compromise between the infinite subject of vortex dynamics and the finite size of the book. Here the author gives a short review of other active areas or points of growth in vortex dynamics. A more detailed presentation of these topics would necessitate the publication of several more new monographs. In chapter 13 the effects of viscosity are set out. The author intentionally avoided this topic in the main part of the book because of his definition of the subject. Here we have the plane exact solutions of the Navier–Stokes equation, results on the decay of trailing vortices, a unique review of Burgers vortices (resulting from a balance between vortex line stretching and diffusion). In chapter 14 there is a review of variational principles of vortex dynamics such as Kelvin's and Arnol'd's variational principles for energy, Hamiltonian dynamics of vorticity, and minimum induced drag for a lifting body. The final section is on vortex breakdown, a popular topic during the last few decades.

Something that a reader who is not acquainted with the subject would not expect is contained in chapters 4 and 5, namely the theory of the motion of a solid body in an ideal fluid. However, the author carefully points out the similarity of this topic to vortex dynamics and considers here a number of general questions such as virtual momentum, the calculation of forces acting on a body, self-propulsion of a deformable body, etc. This topic is especially important in the modern theory of multi-phase media.

The publication of this book by an author who has to a considerable degree helped to create the subject is an important contribution to the teaching and research literature of fluid dynamics. At last this subject is now accessible to a wide circle of students, engineers, teachers, scientists and all those interested in this most intriguing branch of fluid dynamics.

V. A. VLADIMIROV

Nonlinear Random Waves and Turbulence in Nondispersive Media: Waves, Rays, Particles. By S. N. GURBATOV, A. N. MALAKHOV and A. I. SAICHEV (translation edited by D. G. Crighton). Manchester University Press, 1991. 308 pp. £90.

This volume reports important work in the former Soviet Union in nonlinear wave theory and in particular in nonlinear acoustics, a field badly under-represented in Western science. The emphasis is on stochastic problems for first-order scalar hyperbolic equations in one space variable, and in fact for the most part deals with the Riemann equation $v_t + vv_x = 0$, together with the weak shock prescription for discontinuities. Occasionally one-dimensional non-dispersive waves with arbitrary nonlinearity are considered, occasionally the full Burgers equation with non-vanishing diffusivity is discussed, and in the final sections the *vector* Burgers equation is studied in the vanishing diffusivity limit, with application to the distribution of matter on the large scale in the Universe in mind.

Much of the material concerns the relationship between the statistical descriptions of waves in the Eulerian representation (e.g. the correlation function $\langle v(x,t)\,v(x+r, t+\tau)\rangle$ and its Fourier transform, the power spectral density of $\langle v^2\rangle$), which is the description usually required in practice, and the statistics in the Lagrangian representation, for which one has extremely simple evolution, $\mathrm{d}v/\mathrm{d}t = 0$, and an interpretation in terms of the flow of a one-dimensional stream of non-interacting particles. In deterministic problems, it is common in analysis to exploit particular features in particular problems to unravel the Eulerian–Lagrangian transformation; but a complete solution of the Eulerian statistical evolution problem requires one to address the general problem of Eulerian–Lagrangian statistics head-on. Accordingly that topic occupies a significant part of the book, discusses material not readily to hand elsewhere, and should be useful in many other contexts.

For much of the time the field $v(x,t)$ is identified with the velocity field of one-dimensional gas dynamics, or in Lagrangian form as the velocity of a particular one of a row of propagating non-interacting particles. There is, however, frequent mention of the ray optical analogue, in which $v = S_x$, the action S is the wave phase function, and the Hamilton–Jacobi equation $S_t + \frac{1}{2}S_x^2 = 0$ is simply the potential form of the Riemann equation. Then the Hopf–Cole transformation, which linearizes the Burgers equation to the diffusion equation, is the inverse of the transformation frequently employed in optics, which takes the parabolic linear wave equation into the nonlinear Hamilton–Jacobi equation for the wave phase function. Thus there is an analogue between the evolution of gas dynamical disturbances initiated from stochastic conditions and the evolution of optical waves which have passed through a random phase screen.

In the final chapters a very unusual topic of wide interest is analysed by continuum methods – that of the agglomeration of gravitating matter on the large scale in the Universe. One observes in experiment, and in the numerical integration of the differential equations for large numbers of point gravitating masses, the formation of a cellular structure with galaxies in the nodes. Here the authors propose development of a model due originally to Zel'dovich which comprises the vector Burgers equation for velocity $\boldsymbol{v}(\boldsymbol{x}, t)$, a constraint that guarantees the irrotationality of $\boldsymbol{v}(\boldsymbol{x}, t)$, and thereby permits the introduction of the vector Hopf–Cole linearizing transformation $\boldsymbol{v} = -2\nu\nabla \ln \Phi$, and the usual continuity equation for density. From unstructured initial perturbations, a field of non-parallel planar shocks is formed, defining a cellular structure of planes, ribs and nodes. Normal velocities jump at the shocks, tangential velocities remain continuous, and the velocity $\boldsymbol{v}$ is linear in $\boldsymbol{x}$ between shocks. Integration of the continuity equation for such a velocity field shows that the matter is first transferred to the shock planes, then along them to the ribs, and finally to the nodes, in processes for which this book gives an exact statistical description. Comparisons with recent direct numerical simulations of the motion of large numbers of gravitating point masses confirm the essence and accuracy of this description.

This book is a valuable update on J. M. Burgers' own 1974 book *The Nonlinear Diffusion Equation* (Reidel, Dordrecht) and should interest a wide range of readers working in both deterministic and stochastic problems of linear and nonlinear wave propagation, as well as the astrophysical community interested in the large-scale distribution of galactic matter.

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